Homework 10

Solutions!

1. Problem 5.47 from the book (p 252)
   Solution:
   Dependent. For example: $P(Y_1 = 1, Y_2 = 2) \neq P(Y_1 = 1)P(Y_2 = 2)$.

2. Problem 5.52 from the book (p 253)
   Solution:
   Note that $f(y_1, y_2)$ can be factored and the ranges of y_1 and y_2 do not depend on each other so by Theorem 5.5 Y_1 and Y_2 are independent.

3. Problem 5.53 from the book (p 253)
   Solution:
   We know from problem 5.27 that the marginal PDFs of Y_1 and Y_2 are

$$f_1(y_1) = \begin{cases} 3(1-y_1)^2 & 0 < y_1 < 1 \\ 0 & \text{else} \end{cases}$$

and

$$f_2(y_2) = \begin{cases} 6y_2(1-y_2) & 0 < y_2 < 1 \\ 0 & \text{else} \end{cases}$$

respectively. Now consider whether $f(.5, .5) = f_1(.5)f_2(.5)$

$$\begin{array}{rcl} f(.5,.5) & ? & f_1(.5)f_2(.5) \\ 6(.5) & ? & 3(.5)^2 6(.5)^2 \\ 3 & \neq & 18/4 \end{array}$$

Thus, Y_1 and Y_2 cannot be independent. Therefore Y_1 and Y_2 are dependent
Note: An alternative demonstration of the dependence of Y_1 and Y_2 would have been to note that the support of Y_1 is restricted given a value of Y_2 (or vice a versa).

4. Problem 5.73 from the book (p 261)
Solution:

Use the mean of the hypergeometric: $E(Y_1) = 3(4)/9 = 4/3$.

5. Problem 5.77 from the book (p 262)
Solution:

Following Ex. 5.27, the marginal densities can be used:

a. $E(Y_1) = \int_0^1 3y_1(1-y_1)^2\,dy_1 = 1/4$, $E(Y_2) = \int_0^1 6y_2(1-y_2)dy_2 = 1/2$.

b. $E(Y_1^2) = \int_0^1 3y_1^2(1-y_1)^2\,dy_1 = 1/10$, $V(Y_1) = 1/10 - (1/4)^2 = 3/80$,

$E(Y_2^2) = \int_0^1 6y_2^2(1-y_2)dy_2 = 3/10$, $V(Y_2) = 3/10 - (1/2)^2 = 1/20$.

c. $E(Y_1 - 3Y_2) = E(Y_1) - 3 \cdot E(Y_2) = 1/4 - 3/2 = -5/4$.