Homework 9

Solutions!

1. Problem 5.3 from the book (p 232)
   Solution:
   Note that using material from Chapter 3, the joint probability function is given by

   $p(y_1, y_2) = P(Y_1 = y_1, Y_2 = y_2) = \dfrac{\binom{4}{y_1}\binom{3}{y_2}\binom{2}{3-y_1-y_2}}{\binom{9}{3}}$, where $0 \le y_1$, $0 \le y_2$, and $y_1 + y_2 \le 3$.

   In table format, this is

y_2 \ y_1	0	1	2	3
0	0	3/84	6/84	1/84
1	4/84	24/84	12/84	0
2	12/84	18/84	0	0
3	4/84	0	0	0

2. Problem 5.9 from the book (p 233)
   Solution:

   a. Since the density must integrate to 1, evaluate $\int_0^1 \int_0^{y_2} k(1-y_2)dy_1 dy_2 = k/6 = 1$, so $k = 6$.

   b. Note that since $Y_1 \le Y_2$, the probability must be found in two parts (drawing a picture is useful):

   $P(Y_1 \le 3/4, Y_2 \ge 1/2) = \int_{1/2}^{1} \int_{0}^{1/2} 6(1-y_2)dy_1 dy_2 + \int_{1/2}^{3/4} \int_{y_1}^{1} 6(1-y_2)dy_2 dy_1$ =24/64 + 7/64 = 31/64.

3. Problem 5.21 from the book (p 244)
   Solution:

a. The marginal distribution of Y_1 is hypergeometric with $N = 9$, $n = 3$, and $r = 4$.

b. Similar to part a, the marginal distribution of Y_2 is hypergeometric with $N = 9$, $n = 3$, and $r = 3$. Thus,

$$P(Y_1 = 1 \mid Y_2 = 2) = \frac{P(Y_1=1, Y_2=2)}{P(Y_2=2)} = \frac{\binom{4}{1}\binom{3}{2}\binom{2}{0}}{\binom{9}{3}} \Bigg/ \frac{\binom{3}{2}\binom{6}{1}}{\binom{9}{3}} = 2/3.$$

c. Similar to part b,

$$P(Y_3 = 1 \mid Y_2 = 1) = P(Y_1 = 1 \mid Y_2 = 1) = \frac{P(Y_1=1, Y_2=1)}{P(Y_2=1)} = \frac{\binom{3}{1}\binom{2}{1}\binom{4}{1}}{\binom{9}{3}} \Bigg/ \frac{\binom{3}{1}\binom{6}{2}}{\binom{9}{3}} = 8/15.$$

4. Problem 5.27 from the book (p 244)
Solution:

a. $f_1(y_1) = \int_{y_1}^{1} 6(1 - y_2)dy_2 = 3(1 - y_1)^2$, $0 \le y_1 \le 1$;

$f_2(y_2) = \int_{0}^{y_2} 6(1 - y_2)dy_1 = 6y_2(1 - y_2)$, $0 \le y_2 \le 1$.

b. $$P(Y_2 \le 1/2 | Y_1 \le 3/4) = \frac{\int_0^{1/2}\int_0^{y_2} 6(1 - y_2)dy_1 dy_2}{\int_0^{3/4} 3(1 - y_1)^2 dy_1} = 32/63.$$

c. $f(y_1 \mid y_2) = 1/y_2$, $0 \le y_1 \le y_2 \le 1$.

d. $f(y_2 \mid y_1) = 2(1 - y_2)/(1 - y_1)^2$, $0 \le y_1 \le y_2 \le 1$.

e. From part **d**, $f(y_2 \mid 1/2) = 8(1 - y_2)$, $1/2 \le y_2 \le 1$. Thus, $P(Y_2 \ge 3/4 \mid Y_1 = 1/2) = 1/4$.