Homework 3

SOLUTIONS!

1. Problem 2.62 from the book (p 50)
   Solution:
   We can think of this simply in terms of the first production line. There are $\binom{2}{2}\binom{7}{1}$ ways to select three motors, two of which are from the particular supplier. Any order of selecting the motors will work, so there are 3! orders in which we can select the three motors. Similarly, there are $\binom{9}{3}$ ways to select three mototrs for the first line in general, and there are 3! way to order these motors. So, the probability that we select the two motors from the particular supplier for the first production line is:

$$\begin{aligned}
\frac{\#\text{ of ways to fulfill event}}{\#\text{ total possible ways}} &= \frac{\#\text{ of ways to select for event} \times \#\text{ of ways to order}}{\#\text{ of ways to select in general} \times \#\text{ of ways to order}} \\
&= \frac{\binom{2}{2}\binom{7}{1} \times 3!}{\binom{9}{3} \times 3!} \\
&= \frac{7}{9!/(3!6!)} \\
&= \frac{3!7!}{9!} \\
&= \frac{1}{12} \\
&\approx .083
\end{aligned}$$

2. Problem 2.137 from the book (p 75)
   Solution:
   Let A = {both balls are white}, and for i = 1, 2, … 5

   A_i = both balls selected from bowl i are white. Then $\bigcup A_i = A$.

   B_i = bowl i is selected. Then, $P(B_i) = .2$ for all i.

   a. $P(A) = \sum P(A_i \mid B_i)P(B_i) = \frac{1}{5}\left[0 + \frac{2}{5}\left(\frac{1}{4}\right) + \frac{3}{5}\left(\frac{2}{4}\right) + \frac{4}{5}\left(\frac{3}{4}\right) + 1\right] = 2/5$.

   b. Using Bayes' rule, $P(B_3|A) = \frac{\frac{3}{50}}{\frac{2}{50}} = 3/20$.

3. Suppose that we have 4 Bags:

Bag 1:	1 Red Marble	1 Blue Marble	2 Green Marbles
Bag 2:	2 Red Marbles	1 Blue Marble	1 Green Marble
Bag 3:	1 Red Marble	2 Blue Marbles	1 Green Marble
Bag 4:	1 slip of paper with #1	2 slips of paper with #2	1 slip of paper with #3

Suppose we play a game where we perform the following steps:

a) We draw a slip of paper from Bag 4

b) If we draw a slip that says #1 then we draw a marble from Bag 1. If we draw a slip that says #2 then we draw a marble from Bag 2. If we draw a slip that says #3 then we draw a marble from Bag 3.

Given that a red marble is drawn, what is the probability that we drew a slip of paper that says #2 on it?
Solution:
Let B_i be the event that we drew a slip of paper that says $\#i$ on it for $i = 1, 2, 3$ and let A be the event that we draw a red marble. Then by Bayes rule we see that

$$P(B_2|A) = \frac{P(A|B_2)P(B_2)}{\sum_{i=1}^{3} P(A|B_i)P(B_i)}$$

So, to solve for $P(B_2|A)$, we observe and/or calculate the following values:

$$\begin{aligned} P(B_1) &= \frac{1}{4} \\ P(B_2) &= \frac{1}{2} \\ P(B_3) &= \frac{1}{4} \\ P(A|B_1) &= \frac{1}{4} \\ P(A|B_2) &= \frac{1}{2} \\ P(A|B_3) &= \frac{1}{4} \end{aligned}$$

Plugging these values into our Bayes Rule formulation, we find that

$$\begin{aligned}
P(B_2|A) &= \frac{P(A|B_2)P(B_2)}{\sum_{i=1}^{3} P(A|B_i)P(B_i)} \\
&= \frac{(1/2)(1/2)}{(1/4)(1/4) + (1/2)(1/2) + (1/4)(1/4)} \\
&= \frac{1/4}{6/16} \\
&= \frac{4}{6} \\
&= \frac{2}{3}
\end{aligned}$$

4. Let X be a random variable with the following pdf:

$$p_x(x) = (\frac{1}{2})^{x+1} x! \text{ for } x = 0, 1, 2$$

a) Verify that $p_x(x)$ is a valid pdf

Solution:

In order to show that p_x is a valid pdf, we must show that

i. $0 \leq p_x(x) \leq 1 \forall x \in S$, where S is the support for X

ii. $\sum_{x \in S} p_x(x) = 1$

i. $0 \leq p_x(x) \leq 1 \forall x \in S$, where S is the support for X

To show this, we can simply examine $p_x(x) \forall x \in S$, which in our case is $x = 0, 1, 2$

$$\begin{aligned}
p_x(0) &= \frac{1}{2}^{(0+1)} 0! \\
&= \frac{1}{2} \\
p_x(1) &= \frac{1}{2}^{(1+1)} 1! \\
&= \frac{1}{4} \\
p_x(2) &= \frac{1}{2}^{(2+1)} 2! \\
&= \frac{1}{8} 2 \\
&= \frac{1}{4}
\end{aligned}$$

We see that $0 \leq p_x(x) \leq 1 \forall x \in S$, thus the first condition is met

ii. $\sum_{x \in S} p_x(x) = 1$

$$\begin{aligned}\sum_{x \in S} p_x(x) &= \frac{1}{2} + \frac{1}{4} + \frac{1}{4} \\ &= 1\end{aligned}$$

Since both conditions are fulfilled, we see that $p_x(x)$ is a valid pdf.

b) Find $E[X]$
Solution:

$$\begin{aligned}E[X] &= \sum_{x \in S} x p_x(x) \\ &= 0p_x(0) + 1p_x(1) + 2p_x(2) \\ &= 0 + \frac{1}{4} + 2\frac{1}{4} \\ &= \frac{3}{4}\end{aligned}$$

5. Problem 3.19 from the book (p 98)
Solution:
Let P be a random variable that represents the company's profit. Then, $P = C - 15$ with probability 98/100 and $P = C - 15 - 1000$ with probability 2/100. Then, $E(P) = (C - 15)(98/100) + (C - 15 - 1000)(2/100) = 50$. Thus, $C = \$85$.